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WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education
Advanced Subsidiary/Advanced

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MARKING SCHEMES

JANUARY 2007

MATHEMATICS

WJEC
CBAC

INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2007 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

MATHEMATICS C1

1.	(*a*)	Gradient $AC = ½$ (o.e.)	B1
		Gradient $BD = –2$ (o.e.)	B1
		Gradient AC × Gradient $BD = –1$	M1
		$\therefore$ AC and BD are perpendicular	A1
	(*b*)	Gradient $AD = –⅓$ (o.e.)	B1
		Gradient $BC = –⅓$ (o.e.)	B1
		Gradient AD = Gradient BC $\therefore$ AD and BC are parallel	B1
	(*c*)	A correct method for finding the equation of AC (or BD) using candidate's gradient for AC (or BD)	M1
		Equation of AC : $y – 0 = ½ (x + 5)$	
		$x – 2y + 5 = 0$ (convincing)	A1
		Equation of BD : $y + 3 = –2(x – 4)$ (or equivalent)	
		(f.t. candidate's gradient for BD)	A1
		Special case:	
		Verification of equation of AC by substituting coordinates of **both** A and C into the given equation	B1
	(*d*)	(i) An attempt to solve equations of AC and BD simultaneously	M1
		$x = 1, y = 3$ (convincing)	A1
		(ii) A correct method for finding the length of AE	M1
		$AE = \sqrt{45}$	A1
		Special case for (*d*)(i)	
		Substituting (1, 3) in equations of **both** AC and BD	M1
		Convincing argument that coordinates of E are (1, 3)	A1
2.	(*a*)	$2\sqrt{32} + 3\sqrt{8} – \sqrt{18} = 8\sqrt{2} + 6\sqrt{2} – 3\sqrt{2}$ (one correct)	B1
		(another correct)	B1
		$2\sqrt{32} + 3\sqrt{8} – \sqrt{18} = 11\sqrt{2}$ (c.a.o.)	B1
	(*b*)	$\dfrac{6 + \sqrt{30}}{6 – \sqrt{30}} = \dfrac{(6 + \sqrt{30})(6 + \sqrt{30})}{(6 – \sqrt{30})(6 + \sqrt{30})}$	M1
		Numerator: $36 + 30 + 6\sqrt{30} + 6\sqrt{30}$	A1
		Denominator: $36 – 30$	A1
		$\dfrac{6 + \sqrt{30}}{6 – \sqrt{30}} = 11 + 2\sqrt{30}$ (c.a.o.)	A1
		Special case	
		If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $6 – \sqrt{30}$	

3. (*a*) Either: use of $f(1) = 8$
Or: division by $(x - 1)$ leading to $p + k = 8$ M1
A convincing argument that $p = -2$ A1
Special case
Candidates who assume $p = -2$ awarded B1

(*b*) Attempting to find $f(r) = 0$ for some value of r M1
$f(-1) = 0 \Rightarrow x + 1$ is a factor A1
$f(x) = (x + 1)(9x^2 + ax + b)$ with one of a, b correct M1
$f(x) = (x + 1)(9x^2 - 3x - 2)$ A1
$f(x) = (x + 1)(3x + 1)(3x - 2)$ (f.t. one slip) A1
Special case
Candidates who find one of the remaining factors,
$(3x + 1)$ or $(3x - 2)$, using e.g. factor theorem, awarded B1

4. (*a*) $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ (–1 for each error)
(–1 for any subsequent 'simplification') B2
(*b*) $(2 + x)^4 = 2^4 + 4 \times (2)^3 \times x + 6 \times (2)^2 \times x^2 + 4 \times (2) \times x^3 + x^4$
(f.t. for at least 4 terms, not all coefficients equal to 1)
$(2 + x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ (–1 for each error) B2
An attempt to collect terms and form quadratic equation M1
$16 + 32x + 24x^2 + 8x^3 + x^4 = 14 + 33x + 25x^2 + 8x^3 + x^4$
$\Rightarrow x^2 + x - 2 = 0 \Rightarrow (x + 2)(x - 1) = 0 \Rightarrow x = -2, 1$ (c.a.o.) A1

5. (*a*) $y = 2x^2 - 5x + 3$
$y + \delta y = 2(x + \delta x)^2 - 5(x + \delta x) + 3$ B1
Subtracting y from above to find δy M1
$\delta y = 4x\delta x + 2(\delta x)^2 - 5\delta x$ A1
Dividing by δx and letting $\delta x \to 0$ M1
$\frac{dy}{dx} = 4x - 5$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 3$ at $x = 2$ B1
Gradient of normal = $\dfrac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ M1
Equation of normal: $y - 1 = -\tfrac{1}{3}(x - 2)$ (or equivalent) A1
(f.t. candidate's numerical value for $\frac{dy}{dx}$)

6. (*a*) $\frac{dy}{dx} = 10x^4 - 48x^{-3} - \frac{3}{2}x^{-1/2}$ B1, B1, B1

(*b*) $\frac{d}{dx}\{x^2(3x + 1)\} = \frac{d}{dx}(3x^3 + x^2)$ (multiplying and differentiating) M1
$\frac{d}{dx}\{x^2(3x + 1)\} = 9x^2 + 2x$ A1

Special case
Correct use of product formula leading to
$\frac{d}{dx}\{x^2(3x + 1)\} = x^2 \times 3 + 2x \times (3x + 1)$ B2

7. An expression for $b^2 - 4ac$, with $b = \pm 4$, and at least one of a or c correct M1
$b^2 - 4ac = (\pm 4)^2 - 4k(k - 3)$ A1
Putting $b^2 - 4ac \geq 0$ or $b^2 - 4ac > 0$ m1
$k^2 - 3k - 4 \leq 0$ (convincing) A1
Finding fixed points $k = 4$, $k = -1$ (c.a.o.) B1
$-1 \leq k \leq 4$ **or** $4 \geq k \geq -1$ **or** $[-1, 4]$ **or** a correctly worded statement to the effect that k lies between -1 and 4 inclusive
(f.t. candidate's fixed points) B2
Note: $-1 < k < 4$,
$k \leq 4, -1 \leq k$
$k \leq 4$ or $-1 \leq k$
all earn B1

8. (*a*) $a = 2$ B1
$b = 5$ B1
Maximum value $= \frac{1}{5}$ (f.t. candidate's a and b) B2

(*b*) $x + 2 = x^2 - 5x + 11$ M1
An attempt to collect terms, form and solve quadratic equation m1
$(x - 3)^2 = 0 \Rightarrow$ curve and line touching A1
$x = 3, y = 5$ A1
Special case
Differentiating and equating to get $1 = 2x - 5$ M1
$x = 3$ A1
$y = 5$ (from one equation) A1
Verification that $x = 3$, $y = 5$ satisfies other equation A1

9. (*a*) $\frac{dy}{dx} = 12x^2 - 12$ B1
Putting derived $\frac{dy}{dx} = 0$ M1
$x = -1, 1$ (only one correctly derived root required)
(f.t. candidate's $\frac{dy}{dx}$) A1
Stationary points are $(-1, 11)$ and $(1, -5)$ (both correct) (c.a.o.) A1
A correct method for finding nature of stationary points M1
$(-1, 11)$ is a maximum point (f.t. candidate's derived values) A1
$(1, -5)$ is a minimum point (f.t. candidate's derived values) A1

(*b*) Graph

Shape of cubic M1

Stationary points (f.t. candidate's derived points) A1 A1

(*c*) Graph

x-translation ± 1, no y-translation (of graph drawn in (*b*)) M1

Stationary points (f.t. candidate's derived points) A1 A1

MATHEMATICS C2

1. Correct formula with $h = 0{\cdot}25$ M1

1	1·7320508		
1·25	1·9882467		
1·5	2·3184046		
1·75	2·7128168	(3 values correct)	B1
2	3·1622777	(5 values correct)	B1

$I \approx 0{\cdot}125\{1{\cdot}7320508 + 3{\cdot}1622777 + 2(1{\cdot}9882467 + 2{\cdot}3184046 + 2{\cdot}7128168)\}$

$I \approx 2{\cdot}367$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}2$

Correct formula with $h = 0{\cdot}2$ M1

1	1·7320508		
1·2	1·9308029		
1·4	2·1780725		
1·6	2·4690079		
1·8	2·7985711		
2	3·1622777	(all values correct)	B1

$I \approx 0{\cdot}1\{1{\cdot}7320508 + 3{\cdot}1622777 + 2(1{\cdot}9308029 + 2{\cdot}1780725 + 2{\cdot}4690079 + 2{\cdot}7985711)\}$

$I \approx 2{\cdot}365$ (f.t. one slip) A1

2. (*a*) $10\sin^2 x - 3\sin x = 4(1 - \sin^2 x) + 1$

(correct use of $\cos^2 x = 1 - \sin^2 x$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin x$, either by using the quadratic formula or by getting the expression into the form $(a\sin x + b)(c\sin x + d)$, with $a \times c$ = coefficient of $\sin^2 x$ and $b \times d$ = constant m1

$14\sin^2 x - 3\sin x - 5 = 0 \Rightarrow (7\sin x - 5)(2\sin x + 1) = 0$

$\Rightarrow \sin x = \frac{5}{7}, -\frac{1}{2}$ A1

$x = 45{\cdot}6°, 134{\cdot}4°, 210°, 330°$ (45·6°, 134·4°) B1

(210°) B1

(330°) B1

Note:

Subtract 1 mark for each additional root in range, ignore roots outside range.

$\sin x$ = +, –, f.t. for 3 marks, $\sin x$ = –, –, f.t. for 2 marks

$\sin x$ = +, +, f.t. for 1 mark

(*b*) $2x + 30° = 60°, 240°, 420°$ (one value) B1

$x = 15°, 105°$ B1, B1

Note:

Subtract 1 mark for each additional root in range, ignore roots outside range.

Special case: Candidates who take $\tan^{-1}\sqrt{3} = 30°$ and whose final answer is $x = 0°$, $x = 180°$ earn B1

3. (*a*) n'th term $= ar^{n-1}$ B1

$S_n = a + ar + \ldots + ar^{n-2} + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = ar + ar^2 + \ldots ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

$S_\infty = \dfrac{a}{1 - r}$ B1

(*b*) (i) $a + ar = k(ar + ar^2)$ ($k = 2, \frac{1}{2}$) M1

$1 + r = 2r + 2r^2$ A1

An attempt to collect terms, form and solve quadratic equation M1

$2r^2 + r - 1 = 0 \Rightarrow (2r - 1)(r + 1) = 0 \Rightarrow r = \frac{1}{2},$ A1

(ii) $r = \frac{1}{2} \Rightarrow \dfrac{a}{1 - \frac{1}{2}} = 12$

(f.t. candidate's value for r provided $0 < r < 1$) B1

$a = 6$ (f.t. candidate's value for r provided $0 < r < 1$) B1

$S_8 = \dfrac{6}{\frac{1}{2}}\{1 - (\tfrac{1}{2})^8\}$ M1

$S_8 \approx 11{\cdot}95$ (f.t. candidate's derived values of a, r) A1

4. $a + 7d = k(a + 2d)$ ($k = 2, \frac{1}{2}$) M1

$a + 7d = 2(a + 2d)$ A1

$a + 19d = 11$ B1

An attempt to solve simultaneous equations M1

$d = \dfrac{1}{2}$, $a = \dfrac{3}{2}$ (both values needed) (f.t. only for $k = \frac{1}{2}$) A1

5. (*a*) $A(3, -4)$ B1

A correct method for finding the radius M1

Radius = 10 A1

(*b*) (i) A clear attempt to find AB using correct formula M1

$AB = 15$ (f.t. candidate's coordinates for A) A1

$AB = r_1 + r_2 \Rightarrow$ circles touch B1

(ii) A correct method for finding gradient of PB (o.e.) M1

Gradient $PB = -\dfrac{4}{3}$ (o.e.) A1

Gradient of tangent $= \dfrac{3}{4}$ (f.t. candidate's gradient for PB) B1

Equation of tangent is

$y - 4 = \dfrac{3}{4}(x + 3)$ (f.t. candidate's gradient for tangent) B1

Special case for (ii)

$2x + 2y\dfrac{dy}{dx} - 6 + 8\dfrac{dy}{dx} = 0$ (at least 3 terms correct) M1

$\dfrac{dy}{dx} = \dfrac{6 - 2x}{2y + 8}$ (f.t. if M1 awarded) A1

Gradient of tangent $= \dfrac{3}{4}$ (f.t. candidate's $\dfrac{dy}{dx}$) A1

Equation of tangent is

$y - 4 = \dfrac{3}{4}(x + 3)$ (f.t. candidate's gradient for tangent) B1

6. (*a*) $\dfrac{1}{2} \times 10 \times 6 \times \sin B\hat{A}C = 15\sqrt{3}$ (use of correct area formula in equation to find $\sin B\hat{A}C$) M1

$\sin B\hat{A}C = \dfrac{\sqrt{3}}{2}$ (o.e.) A1

$B\hat{A}C = 120°$ (2·094 radians) (f.t. candidate's value for $\sin B\hat{A}C$) A1

(*b*) Correct use of cosine formula in equation to find BC M1

$BC^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \times \cos 120°$ (f.t. candidate's $B\hat{A}C$) A1

$BC = 14$ (f.t. one slip) A1

Note: For $B\hat{A}C = 60°$, $BC^2 = 76$, $BC \approx 8{\cdot}72$

7. (*a*) $\dfrac{x^{3/2}}{3/2} + 2\dfrac{x^{-1}}{-1}$ $(+c)$ B1,B1

(*b*) (i) $x^2 + 3 = 4x$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = 3$ m1

$(x - 3)(x - 1) = 0 \Rightarrow x = 1, x = 3 \Rightarrow A(1, 4), B(3, 12)$ A1

(ii)

Either:

Total area $= \int_1^3 4x\,dx - \int_1^3 (x^2+3)\,dx$ (use of integration) M1

(subtracting integrals) m1

$= [2x^2 - \tfrac{1}{3}x^3 - 3x]_1^3$ (correct integration) B3

$= 18 - 9 - 9 - 2 + \tfrac{1}{3} + 3$ (use of candidate's x-values as limits) M1

$= 1\tfrac{1}{3}$ (c.a.o.) A1

Or:

Area of trapezium = 16

(f.t. candidate's coordinates for A, B) B1

Area under curve $= \int_1^3 (x^2+3)\,dx$ (use of integration) M1

$= [\tfrac{1}{3}x^3 + 3x]_1^3$ (correct integration) B2

$= 9 + 9 - \tfrac{1}{3} - 3$ (use of candidate's x-values as limits) M1

$= 14\tfrac{2}{3}$

Finding total area by subtracting values m1

Total area $= 16 - 14\tfrac{2}{3} = 1\tfrac{1}{3}$ (c.a.o.) A1

8. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between p and $\log_a x$) B1

$xy = a^p \times a^q = a^{p+q}$ (the laws of indicies) B1

$\therefore \log_a xy = p + q = \log_a x + \log_a y$ (convincing) B1

(*b*) $\tfrac{1}{2}\log_a 256 = \log_a 256^{\frac{1}{2}}$, $2\log_a 48 = \log_a 48^2$ (one use of power law) B1

$\log_a 36 + \tfrac{1}{2}\log_a 256 - 2\log_a 48 = \log_a \dfrac{36 \times 16}{48^2}$ (addition law) B1

(subtraction law) B1

$\log_a 36 + \tfrac{1}{2}\log_a 256 - 2\log_a 48 = \log_a \tfrac{1}{4}$ B1

(*c*) **Either:**

$(x+1)\ln 2 = \ln 5$ (taking logs on both sides) M1

$x = \dfrac{\ln 5 - \ln 2}{\ln 2}$

$x \approx 1{\cdot}322$ A1

Or:

$(x+1) = \log_2 5$ M1

$x \approx 1{\cdot}322$ A1

9. (*a*) Setting up equation, including the use of arc length = $r\theta$ M1

$2 \times 3 + 3\theta = 10$ A1

$\theta = \frac{4}{3}$ A1

(*b*) Area of sector = $\frac{1}{2} \times 3^2 \times \theta$ (or candidate's value for θ) B1

Area of triangle = $\frac{1}{2} \times 3^2 \times \sin\theta$ (or candidate's value for θ) B1

Use of: area of segment = area of sector – area of triangle
with numerical values on R.H.S. M1

Area of segment $\approx 1{\cdot}626$ (c.a.o.) A1

MATHEMATICS C3

1.	*(a)*	$h = 0{\cdot}2$	M1 (correct formula $h = 0.2$)
		Integral $\approx \dfrac{0\cdot 2}{3}$ [0·69314718 + 1·44456327	
		+ 4 (0·89199804 + 1·26976055)	B1 (3 values)
		+ 2(1·08518927)]	B1 (2 values)
		= 0·864	A1 (F.T. one slip)
	(b)	Second integral ≈ 0·432	B1 (F.T. answer in *(a)*)
			5
2.	*(a)*	$\theta = 0$	B1 (appropriate choice of θ)
		l.h.s. = 1	B1 (*l.h.s* ≠ *r.h.s.*)
		r.h.s. = – 1	
		$(\therefore \cos 3\theta \neq 3\cos^3\theta - 4\cos\theta)$	
	(b)	$\sec^2\theta - 1 + 2\sec\theta = 7$	M1 ($\tan^2\theta = \sec^2\theta - 1$)
		$\sec^2\theta + 2\sec\theta - 8 = 0$	M1 (correct formula for
			for $(a\sec\theta + b)(c\sec\theta + d)$
		$(\sec\theta + 4)(\sec\theta - 2) = 0$	where ac = coeff of $\sec^2\theta$
			bd = constant)
		$\sec\theta = -4, 2$	
		$\cos\theta = -\dfrac{1}{4}, \dfrac{1}{2}$	A1 (C.A.O.)
		θ = (104º – 105º) (255º – 256º)	B1 (104 – 105) B1 (255 – 256)
		60º, 300º	B1 (60º, 300º)
			8

3.

x	$\cos x + 2x - 2$
0	-1
$\dfrac{\pi}{2}$	1·14

M1(attempt to find values (or signs))

Change of sign indicates presence of root (in $(0, \frac{\pi}{2})$) — A1 (correct values (or signs) and conclusion)

$x_0 = 0{\cdot}5$, $x_1 = 0{\cdot}5612087$, $x_2 = 0{\cdot}5766937$ — B1 (x_1)

$x_3 = 0{\cdot}5808650$, $x_4 = 0{\cdot}5820059$ — B1 (x_4)

Root $\approx 0{\cdot}582$

Check $x = 0{\cdot}5815$, $0{\cdot}5828$

x	$\cos x + 2x - 2$
0·5815	-0·0014
0·5825	0·00009

M1(attempt to find values (or signs))

A1 (correct)

Change of sign indicates that the root is 0·582 (correct to 3 decimal places) — A1 (F.T. one slip)

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4. *(a)* $15(1 + 2x)^{14}.2 = 30(1 + 2x)^{14}$

M1 ($15(1 + 2x)^{14}.k$, any k)
A1 ($k = 2$, simplified result)

(b) $\dfrac{1}{1+x^2}.2x\left(=\dfrac{2x}{1+x^2}\right)$

M1 $\left(\dfrac{1}{1+x^2}\times f(x),\ f(x)=1,2,kx\right)$
A1($f(x) = 2x$)
(Final answer)

(c) $\dfrac{(1+\sin x)(-\sin x)-(2+\cos x)\cos x}{(1+\sin x)^2}$

M1 $\left(\dfrac{(1+\sin x)f(x)-(2+\cos x)g(x)}{(1+\sin x)^2}\right)$

A1 ($f(x) = -\sin x$
$g(x) = \cos x$)

$= \dfrac{-1-\sin x-2\cos x}{(1+\sin x)^2}$ — A1 (simplified answer)

(d) $\dfrac{1}{1+(3x)^2}\times 3\ \left(=\dfrac{3}{1+9x^2}\right)$

M1 $\left(\dfrac{k}{1+(3x)^2}, \text{any } k\right)$
A1 ($k = 3$ and final result)

(e) $x^2(\sec^2 x) + (2x)\tan x$

M1 ($x^2 f(x) + g(x)\tan x$)
A1 ($f(x) = \sec^2 x$, $g(x) = \tan x$)

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5. $\left(\frac{dy}{dx}=0\right)$ $2e^{2x}-1=0$ M1 (attempt to find $\frac{dy}{dx}$ and set = 0)

M1 ($ke^{2x}-1$, any k)
A1 ($k = 2$)

$e^{2x}=\frac{1}{2}$

$x=\frac{1}{2}\ln\left(\frac{1}{2}\right)$ (o.e.) A1 (C.A.O.)

$\frac{d^2y}{dx^2}=4e^{2x}$ M1 (correct attempt to use any method)

$\frac{d^2y}{dx^2}=4e^{2x}=2>0$

$\therefore$ minimum point. A1 (F.T. $\frac{d^2y}{dx^2}=ke^{2x}$)

6

Alternative:
Sign test for stationary point

$x=-0{\cdot}34$, $\frac{dy}{dx}=0\cdot 013>0$

$x=-0{\cdot}35$, $\frac{dy}{dx}=-0\cdot 006<0$

$\frac{dy}{dx}$ changes from – to +

$\therefore$ minimum point

6. *(a)* $3x^2+x^2\frac{dy}{dx}+2xy+4y^3\frac{dy}{dx}=0$ B1 ($x^2\frac{dy}{dx}+2xy$)

B1 ($4y^3\frac{dy}{dx}$)

$\frac{dy}{dx}=-\frac{3x^2+2xy}{x^2+4y^3}$ B1 (all correct, for final result C.A.O.)

(b)	(i)	$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2t}{3t^2}\left(=\frac{2}{3t}\right)$		M1 ($\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$)
				A1 (one differentiation) A1 (other differentiation)
		$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{-2}{3t^2}}{3t^2}$		M1 (correct formula)
				A1 (correct differentiation) F.T. one slip for differentiation of equivalent difficulty)
		$= -\frac{2}{9t^4}$		(o.e.) A1 (C.A.O.)
				9

7.	(a)	(i)	$-\frac{1}{8(2x+3)^4}$	(+C)	M1 $\left(\frac{k}{(2x+3)^4}\text{, any } k\right)$
					A1 $\left(k = -\frac{1}{8}\right)$
		(ii)	$-\frac{1}{3}e^{2-3x}$	(+C)	M1 (ke^{2-3x}, any k)
					A1 ($k = -\frac{1}{3}$)
	(b)	$[2\ln(3x+2)]_0^2$			M1 ($k$ln (3x + 2, any k) A1 (k = 2)
		=	2 (ln 8 – ln 2)		A1 (k (ln 8 – ln 2, F.T. previous k)
		=	2 ln 4		
		=	ln 16		A1 (F.t., k allow ln $\left(\frac{8}{2}\right)^k$)
	(c)	$\left[\frac{1}{3}\sin\left(3x+\frac{\pi}{4}\right)\right]_0^{\frac{\pi}{4}}$			M1 (k sin $\left(3x+\frac{\pi}{4}\right)$, any +ve k)
					A1 ($k = \frac{1}{3}$)
		=	$\frac{1}{3}\left[\sin\pi - \sin\frac{\pi}{4}\right]$		A1 (k(sinπ – sin $\frac{\pi}{4}$)
					F.T. previous k
		=	$-\frac{1}{3\sqrt{2}}$	(o.e.)	A1 (C.A.O.)
					12

8.	First graph	B1 ($y = -4$ for stationary pt)
		B1 (shift of 3 to right, 2 correct x values)
		B1 (all correct) C.A.O.
	Second graph	M1 (maximum in second quadrant)
		A1 (correct intercept at (0, 1))
		A1 (correct st pt)
		6

9.	*(a)*	Let $y = \ln(5x - 4) + 2$	B1 (attempt to isolate x, $y - 2 = ...$)
		$y - 2 = \ln(5x - 4)$	
		$e^{y-2} = 5x - 4$	M1 (exponentiating)
		$x = \dfrac{e^{y-2} + 4}{5}$	A1
		$f^{-1}(x) = \dfrac{e^{x-2} + 4}{5}$	A1
			$\left\{ \text{F.T. one slip} \right\}$
	(b)	domain $[2, \infty)$, range $[1, \infty)$	B1, B1
			6

10. $|\,2x + 1\,| + 2 + 5 > 10$ M1 (attempt at composition, correct order)

$x > 1$ B1

$2x + 1 < -3$ M1 ($2x + 1 < -3$)

$x < -2$ A1

$x > 1$ or $x < -2$ or $(1, \alpha) \cup (-2, -2)$ A1

For incorrect composition,

$\|\,2(x + 5) + 1\,\| + 2 > 10$	M0
$x > -\dfrac{3}{2}$	B0
$\|2x + 11\,\| < -8$	M1
$x < -\dfrac{19}{2}$	A1 (F.T.)
$x > -\dfrac{3}{2}$ or $x < -\dfrac{19}{2}$ or $\left(-\dfrac{3}{2}, \infty\right) \cup \left(-\infty, -\dfrac{19}{2}\right)$	A1 (F.T.)

Alternatively,

$\|\,2x + 1\| > 3$ $(2x + 1)^2 > 9$	M1
$x^2 + x - 2 > 0$ $(x + 2)(x - 1) > 0$ $x > 1$	B1
$x < -2$	M1, A1
$x > 1$ or $x < -2$ or $(1, \infty) \cup (-\infty, -2)$	A1

5

MATHEMATICS FP1

1 $$\sum_{r=1}^{n} r(r+1)(2r+1) = 2\sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$$ B1

$$= \frac{2n^2(n+1)^2}{4} + \frac{3n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$ M1A1

$$= \frac{n(n+1)}{2}(n^2 + n + 2n + 1 + 1)$$ A1

$$= \frac{n(n+1)^2(n+2)}{2}$$ A1

2 (a) Det = 1(3.2 – 4.1) – 2(2.2 – 3.1) + 1(2.4 – 3.3)

= -1 M1A1 Cof matrix = $\begin{bmatrix} 2 & -1 & -1 \\ 0 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$

M1A1

Inverse = $\begin{bmatrix} -2 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$ M1A1

(b) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$ M1

$$= \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$$ A1

3 (a) $$\frac{(3+4\text{i})(1+2\text{i})}{1+3\text{i}} = \frac{(-5+10\text{i})}{1+3\text{i}}$$ B1

$$= \frac{(-5+10\text{i})(1-3\text{i})}{(1+3\text{i})(1-3\text{i})}$$ M1

$$= \frac{25+25\text{i}}{10}$$ A1A1

$$(= \frac{5}{2} + \frac{5}{2}\text{i})$$

(b) $$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2); \arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$ B1

(b) arg(3 + 4i) + arg(1 + 2i) – arg(1 + 3i) = $\arg\left(\frac{5}{2}+\frac{5}{2}\mathrm{i}\right)$ M1

giving $\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}2-\tan^{-1}3=\frac{\pi}{4}$ A1

4 The proposition is true for $n = 1$ since $6^1+4=10$ is divisible by 5. B1
Assume the proposition is true for $n = k$, that is
6^k+4 is divisible by 5 or $6^k+4=5N$ M1
Consider
$6^{k+1}+4=6^k.6+4$ M1
$= 6(5N-4)+4$ A1
$= 30N-20$ A1
Each of the two terms is divisible by 5 so therefore is the left hand side. A1
So, if the proposition is true for $n = k$, it is also true for $n = k + 1$. Since we have shown it to be true for $n = 1$, the proposition is proved by induction. A1

5 (a) Using reduction to echelon form,
$x+2y-z=2$ M1
$5y-3z=1$ A1
$15y-9z=3$ A1
The equations are consistent because the 2nd and 3rd equations are effectively the same equation. A1
(b) Put $z = \alpha$ M1
$y=\frac{1+3\alpha}{5}$ A1
$x= 2+\alpha-\frac{2}{5}(1+3\alpha)=\frac{8-\alpha}{5}$ A1

6 (a) $\ln f(x) = -[\ln(x)]^2$ M1
$\frac{f'(x)}{f(x)}=-2\ln x.\frac{1}{x}$ A1A1
At the stationary point,
$f'(x)=0$ M1
so $x = 1$ and $y = 1$ A1
(b) We now need to determine its nature.
We see from above that
For $x < 1$, $f'(x)>0$ and for $x > 1$, $f'(x)<0$ M1
Showing it to be a maximum. A1

7 $\alpha+\beta+\gamma=-2, \beta\gamma+\gamma\alpha+\alpha\beta=3, \alpha\beta\gamma=4$ B1

Consider

$$\frac{\beta\gamma}{\alpha}+\frac{\gamma\alpha}{\beta}+\frac{\alpha\beta}{\gamma}=\frac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{\alpha\beta\gamma}=\frac{(\beta\gamma+\gamma\alpha+\alpha\beta)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha\beta\gamma}$$ M1A1A1

$$=\frac{25}{4}$$ A1

$$\frac{\gamma\alpha}{\beta}.\frac{\alpha\beta}{\gamma}+\frac{\alpha\beta}{\gamma}.\frac{\beta\gamma}{\alpha}+\frac{\beta\gamma}{\alpha}.\frac{\gamma\alpha}{\beta}=\alpha^2+\beta^2+\gamma^2$$ M1

$$=(\alpha+\beta+\gamma)^2-2(\beta\gamma+\gamma\alpha+\alpha\beta)$$ A1

$$=-2$$ A1

$$\frac{\beta\gamma}{\alpha}.\frac{\gamma\alpha}{\beta}.\frac{\alpha\beta}{\gamma}=\alpha\beta\gamma=4$$ M1A1

The required cubic equation is therefore

$$x^3-\frac{25}{4}x^2-2x-4=0$$ B1

8 (a)

$$\mathbf{T_1}=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$ B1

$$\mathbf{T_2}=\begin{bmatrix}1 & 0 & h\\ 0 & 1 & k\\ 0 & 0 & 1\end{bmatrix}$$ B1

$$\mathbf{T}=\begin{bmatrix}1 & 0 & h\\ 0 & 1 & k\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$ M1

$$=\begin{bmatrix}\cos\theta & -\sin\theta & h\\ \sin\theta & \cos\theta & k\\ 0 & 0 & 1\end{bmatrix}$$ A1

(b) We are given that

$-\sin\theta + h = 1$ M1

$\cos\theta + k = 2$

$3\cos\theta + h = 4$

$3\sin\theta + k = 3$ A1

1st and 4th equations give $3 - k = 3h - 3$ M1

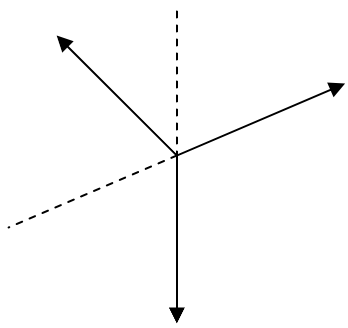
2nd and 3rd equations give $4 - h = 6 - 3k$ A1

The solution is $h=\frac{8}{5}, k=\frac{6}{5}$. M1A1

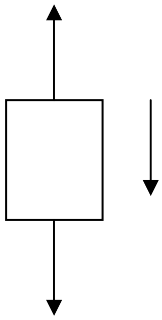
Also, $\sin\theta = 3/5$ and $\cos\theta = 4/5$ giving $\theta = 37°$. A1

9	(a)	Put $z = x + \mathrm{i}y$.	M1
		$\lvert x - 3 + \mathrm{i}y \rvert = \lvert x + \mathrm{i}(y+1) \rvert$	A1
		$(x-3)^2 + y^2 = x^2 + (y+1)^2$	A1
		$x^2 - 6x + 9 + y^2 = x^2 + y^2 + 2y + 1$	A1
		$3x + y = 4$	A1
	(b)	The condition is equivalent to $x^2 + y^2 = 16$.	M1
		Substituting,	
		$x^2 + (4 - 3x)^2 = 16$	M1
		$10x^2 - 24x = 0$	A1
		$x = 0,\ y = 4$	M1A1
		$x = \dfrac{12}{5}, y = -\dfrac{16}{5}$	M1A1

MATHEMATICS M1

1.(a)	Using $v^2 = u^2 + 2as$ with $v = 0$, $u = 10.5$, $a = (-)9.8$		M1
	$0 = 10.5^2 - 2 \times 9.8s$		A1
	$s = \underline{5.625 \text{ m}}$		A1
1.(b)	Using $s = ut + 0.5at^2$ with $t = 5$, $u = 10.5$, $a = (-)9.8$		M1
	$s = 10.5 \times 5 - 0.5 \times 9.8 \times 5^2$		A1
	$s = -70$		A1
	Height of cliff is $\underline{70\text{m}}$		
2.(a)	$T = \underline{30\text{g}} = (\underline{294 \text{ N}})$		B1
2.(b)			B1
2.(c)	Resolve 'horizontally' to obtain equation		M1
	$T_1 \sin 45^\circ = T_2 \sin 60^\circ$		A1 B1
	$\frac{T_1}{\sqrt{2}} = T_2 \sqrt{\frac{3}{2}}$		
	Resolve 'vertically' to obtain equation		M1
	$T_1 \cos 45^\circ + T_2 \cos 60^\circ = 294$		A1
	$T_2 \sqrt{\frac{3}{2}} \frac{1}{\sqrt{2}} + \frac{1}{2} T_2 = 294$		m1
	$(1 + \sqrt{3})\ T_2 = 294 \times 2$		
	$T_2 = 215.223 = 215 \text{ N}$	cao	A1
	$T_1 = 215.223 \times \sqrt{\frac{3}{2}} = 264 \text{ N}$	cao	A1

3.(a)



N2L $\quad 5600g - T = 5600a \quad$ dim. correct $\quad$ M1

$$a = \frac{5600 \times 9.8 - 50400}{5600} \qquad \text{A1}$$

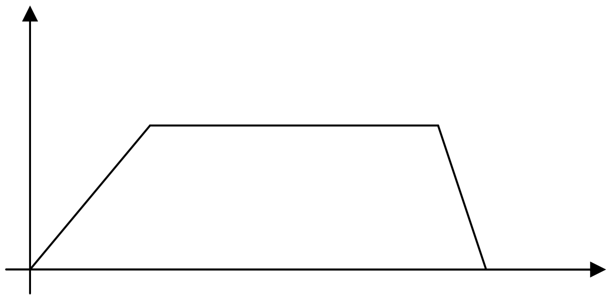
$a = 0.8\text{ms}^{-2}$

3.(b) Using $V = u + at$ with $u = 0$, $a = 0.8$, $t = 8$ $\quad$ M1

$V = 0.8 \times 8$

$V = 6.4 \text{ ms}^{-1}$ $\quad$ A1

3.(c)



M1 A1 A1

3.(d) Distance S = area under graph $\quad$ M1

$S = 0.5(25 + 40) \times 6.4$ $\quad$ any correct area $\quad$ B1

$S =$ <u>208 m</u> $\quad$ A1

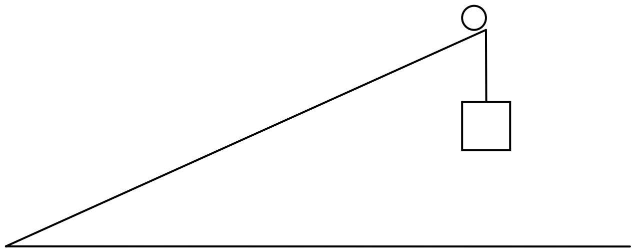
3.(e)

We require $a = -\dfrac{6.4}{(40 - 8 - 25)} = -\dfrac{6.4}{7}$ $\quad$ B1

Therefore Max $T = 5600\left(9.8 + \dfrac{6.4}{7}\right)$ $\quad$ M1

Max $T =$ <u>60000 N</u> $\quad$ A1

4.



	N2L applied to B	M1
	$9g - T = 9a$	A1
	N2L applied to A, weight resolved	M1
	$T - 5g \sin \alpha = 5a$	A1
	Adding $\quad 9g - 5 \times 0.21g = 14a$	m1
	$a = \dfrac{7.95 \times 9.8}{14} = \underline{5.565\,\text{ms}^{-2}}$	A1
	$T = 9(9.8 - 5.565) = \underline{38.115\text{ N}}$	A1
5.(a)	Using $v^2 = u^2 + 2as$ with $v = 0$, $u = 9$, $s = 75$	M1
	$0 = 9^2 + 2 \times 75a$	A1
	$a = \underline{-0.54\text{ ms}^{-1}}$	A1
5.(b)	Using $s = 0.5(u + v)t$ with $v = 0$, $u = 9$, $s = 75$	M1
	$75 = 0.5(0 + 9)t$	A1
	$t = 16\dfrac{2}{3}$	A1
5.(c)	$R = 80g = (784\text{ N})$	B1
	$F = 80 \times 0.54 = (43.2\text{ N})$	M1 A1
	$\mu = \dfrac{F}{R} = \underline{0.055}$ (to 2 sig.figs.)	M1 A1

6.

(a) $I = 2(6 + 4)$ M1

$I = \underline{20 \text{ Ns}}$ A1

(b) Conservation of momentum M1

$12 + 5u = -8 + 5v$ A1

$v - u = 4$

Restitution M1

$v + 4 = -0.75(u - 6)$ A1

$4v + 3u = 2$

Solving simultaneously m1

$4v - 4u = 16$

$4v + 3u = 2$

$7u = -14$

$u = \underline{-2 \text{ ms}^{-1}}$ cao A1

$v = \underline{2 \text{ ms}^{-1}}$ cao A1

8.

(a) Moment about P all forces, dim cor eq. M1

$40g \times 0.8 + 70g \times 2.3 = R_Q \times 1.4$ A1 B1

$R_Q = \underline{1351 \text{ N}}$ A1

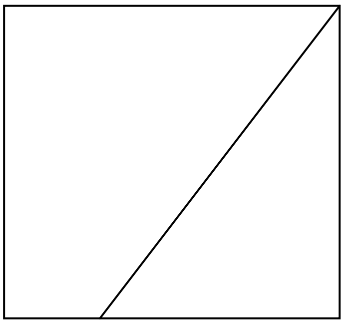
Resolve vertically $R_P + R_Q = 45g + 40g + 70g$ M1

$R_P = \underline{168 \text{ N}}$ A1

(b) If A leaves the bench, the bench would tip about Q as it cannot remain in equilibrium with B at end Y. This is because clockwise moment is greater then anti-clockwise moment.

B1 R1

8.



(a)

	Area	from $AB(x)$	from AE (y)	
$ABCD$	81	4.5	4.5	B1
CDE	27	$3 + 6 \times 2/3$	$9 \times 1/3$	B1
$ABCE$	54	$\bar{x}$	$\bar{y}$	

Moments about AB — M1

$81 \times 4.5 = 27 \times 7 + 54\,\bar{x}$ — ft c's values — A1

$\bar{x} = \underline{3.25 \text{ cm}}$ — cao — A1

Moments about AE — M1

$81 \times 4.5 = 27 \times 3 + 54\,\bar{y}$ — ft c's values — A1

$\bar{y} = \underline{5.25 \text{ cm}}$ — cao — A1

(b) $\theta = \tan^{-1}\left(\dfrac{9-\bar{y}}{9-\bar{x}}\right)$ — correct triangle — M1

$\theta = \tan^{-1}\left(\dfrac{15}{23}\right)$ — ft x, y — A1

$\theta = 33.1^{\circ}$ — ft x, y — A1

MATHEMATICS S1

1	(a)	Prob = $\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}=\frac{1}{21}$ or ${}^4C_3 \div {}^9C_3$	M1A1
	(b)	Prob = $\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7}=\frac{5}{21}$ or ${}^6C_3 \div {}^9C_3$	M1A1
	(c)	Prob = $\frac{2}{9}\times\frac{3}{8}\times\frac{4}{7}\times 6=\frac{2}{7}$ or ${}^2C_1\times{}^3C_1\times{}^4C_1\div{}^9C_3$	M1A1A1
2	(a)	$P(A\cup B)=0.48+0.38-0.28$	M1
		$=0.58$	A1
	(b)	$P(A'\cap B')=1-0.58$	M1
		$=0.42$	A1
	(c)	$P(B\cap A')=0.1$	B1
		$P(A')=0.52$	B1
		$P(B\vert A')=\frac{P(B\cap A')}{P(A')}$	M1
		$=\frac{5}{26}(0.192)$	A1
3		Mean = $n\times 0.1$	B1
		SD = $\sqrt{n\times 0.1\times 0.9}$	B1
		$n\times 0.1=\sqrt{n\times 0.1\times 0.9}$	M1
		$0.01n^2=0.09n$	A1
		$n=9$ (cao)	A1
4	(a)	Choose 2, Prob all heads = 1/4	M1
		Choose 3, Prob all heads = 1/8	
		Choose 4, Prob all heads = 1/16	A1
		P(All heads) = $\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{8}+\frac{1}{3}\times\frac{1}{16}$	M1
		$=\frac{7}{48}$	A1
	(b)	P(2 $\vert$ All heads) = $\frac{1/12}{7/48}$	B1B1
		$=\frac{4}{7}$	B1
		[In (b) award B1 for each correct number]	

5 (a)(i) X is B(20, 0.35) (si) B1

$P(X=5) = \binom{20}{5} \times 0.35^5 \times 0.65^{15}$ M1

(or 0.2454 – 0.1182 or 0.8818 – 0.7546)

$= 0.1272$ A1

(ii) $P(X < 8) = 0.601$ M1A1

(b)(i) Y is B(500, 0.03) and therefore approx Po(15). B1

$P(Y=10) = e^{-15}.\frac{15^{10}}{10!} = 0.0486$ M1A1

(or 0.1185 – 0.0699 or 0.9301 – 0.8815)

(ii) $P(Y > 12) = 0.7324$ M1A1

6 (a) Sum of probs = 1 so

$p + q = 1 - 0.45 = 0.55$ B1

(b) $E(X) = 0.3 + 2p + 0.3 + 4q + 0.25 = 2.75$ M1A1

So $2p + 4q = 1.9$ A1

Substituting from 1st equation into 2nd, M1

$2(0.55 - q) + 4q = 1.9 \rightarrow q = 0.4$ and so $p = 0.15$ AG

[For candidates who simply verify that E(X) = 2.75 with the given values of p and q, award M1A1]

(c) $E(X^2) = 1 \times 0.3 + 4 \times 0.15 + ... + 25 \times 0.05$ M1

$= 9.45$ A1

$\text{Var}(X) = 9.45 - 2.75^2 = 1.8875$ A1

(d) (i) $E(Y) = 4 \times 2.75 + 2 = 13$ M1A1

$\text{Var}(Y) = 16 \times 1.8875 = 30.2$ M1A1

(ii) $P(Y < 15) = P(X < 3.25) = 0.55$ M1A1

7 (a)(i) $E(X) = \int_0^1 20(x^3 - x^4).x\text{d}x$ M1A1

$= 20\left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1$ A1

$= \frac{2}{3}$ A1

(b)(i) $F(x) = \int_0^x 20y^3(1-y)dy$ M1

(limits not required here)

$= 20\left[\frac{y^4}{4} - \frac{y^5}{5}\right]_0^x$ A1

$= 5x^4 - 4x^5$ A1

(ii) Reqd prob $= F(0.6) - F(0.4)$ or $\int_{0.4}^{0.6} 20x^3(1-x)\text{d}x$ M1

$= 5 \times 0.6^4 - 4 \times 0.6^5 - 5 \times 0.4^4 + 4 \times 0.4^5$ A1

$= 0.25$ (cao) A1

(iii)The upper quartile satisfies

$$5q^4 - 4q^5 = \frac{3}{4}$$ M1A1

$$16q^5 - 20q^4 + 3 = 0$$ AG

8	(a)	$\mathrm{P}(X=3) = \mathrm{e}^{-3.75} \times \dfrac{3.75^3}{3!} = 0.207$	M1A1
	(b)(i)	$\mathrm{P}(X \geq 5) = 0.0959$	M1A1
	(ii)	Using the Poisson table 'backwards',	M1
		$\mu = 3.2$	A1
	(c)(i)	P(No errors on 1 page) = $\mathrm{e}^{-0.6}$	M1
		Reqd prob = $\left(\mathrm{e}^{-0.6}\right)^n = \mathrm{e}^{-0.6n}$	A1
	(ii)	We require	
		$\mathrm{e}^{-0.6n} < 0.01$ or $\mathrm{e}^{0.6n} > 100$	
		EITHER	
		Proceed by trial and error.	M1
		For $n = 7$, $\mathrm{e}^{0.6n} = 66.7...$ or $\mathrm{e}^{-0.6n} = 0.014...$	A1
		For $n = 8$, $\mathrm{e}^{-0.6n} = 121.5...$ or $\mathrm{e}^{-0.6n} = 0.0082...$	A1
		Minimum $n = 8$	A1
		OR	
		Solve $0.6n = \ln 100$ (oe)	M1A1
		$n = \dfrac{\ln 100}{0.6} = 7.67..$	A1
		Minimum $n = 8$	A1

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